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THE MATHEMATICS TEACHER

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MATHEMATICS CONTESTS.*

BY ERNEST H. KOCH, JR.

From time immemorial games have been a necessary complement to the more austere forms of intellectual and social development. We have historic records of contests which enable us to obtain a familiar acquaintance with the manners and customs of nations as well as their achievements. To some of us these ancient games were only exhibitions of physical prowess while to others the staging of the orators and actors was a surpassing achievement. In our day we see these contest ideas culminating in great world expositions, art exhibitions, commercial shows and last but not least in the community masques. Cities vie with cities in resetting the contest motive so that each successful affair surpasses its predecessor in beauty and magnificence. International world fairs have permeated the nations, states, municipalities, colleges and schools with the spirit of the contest.

It is our purpose to show how the idea of a school contest has taken an intellectual form although modeled and staged after the physical counterparts on the track and field.

> "What dire offence from amorous causes springs,
> What mighty contests rise from trivial things!"

In the class room we may set apart a definite period during which the pupils may participate in various games, such as checkers, chess or any of the many so-called children's games. These are intended more for the individual than for the group. All games are events of skill in the manipulation of fixed pieces, cards, devices or even individuals in which the element of distribution or motion is only partially under mental control owing to the limitations imposed by the rules of the game. A scholastic contest is a matching of intellects in a set task in which the written or verbal arts of expression play a minor rôle. All contests have their stimulus in the award of medals, banners and the publicity of honors. Any school subject may be used to afford material for a contest. In a commercial school the commercial subjects should have the greatest prominence. A Three "R" Contest is a competitive exhibition in the fundamentals of commercial education. This unique form of school activity was instituted for the purpose of interesting the public and the parents in the work of the school and for promoting the interest of the pupils in those studies which form the foundation of their life work. A program of the events of such a contest follows:

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* Presented at the Baltimore meeting of the Association, December 2, 1916.

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Relay—four fundamental operations combined
6. Scissors—a crisscross in addition
7. Commercial geography
8. Spelling
9. Current events
10. Penmanship
11. Typewriting
12. Stenography

Such a program will give entertainment for two hours and offers the best reply and refutation to that part of the public which so glibly accuses the public schools of graduating poorer material than was graduated years ago. It should be observed that the mathematics takes up half of the program. These programs are altered by substitution of other subjects in the course, such as algebra, geometry, interest, bookkeeping, letter-writing, Spanish and public speaking. Contests take various forms, depending upon the method of public exhibition and demonstration and also upon the special requirements in the preliminary contests of elimination.

In the March, 1916, number of the MATHEMATICS TEACHER and June, 1916, number of *School Science and Mathematics* Mr. John H. McCormick and I have presented an account of a mathematics relay which has been tried in many secondary and elementary schools. The speaker who follows me will illustrate another form of relay which was originated and introduced independently in the William Penn High School of Philadelphia. The latter has been applied to both arithmetic and algebra. One year ago I learned of another independent contribution which was introduced in the Chicago schools. It is evident that these spontaneous and independent efforts in mathematical contests obey the same law of evolution which has given simultaneous and independent birth to identical inventions and to other forms of contemporary expressions of scientific, social and economic thought.

> "Great contest follows, and much learned dust
> Involves the combatants; each claiming truth,
> And truth disclaiming both."

With these general introductory remarks I shall proceed to describe the mathematics relay as it has been developed at the High School of Commerce in New York City. A relay consists of four related parts presented in the following order: a continued addition, a continued subtraction, a continued multiplication and a long division. Four pupils constitute a team. Each member of the team performs and completes one of the four fundamental operations described above. This will be illustrated at the blackboard to make the situation and the performance clear. (Through the courtesy of the Eastern High School of Baltimore several teams of girls demonstrated the relay.) In the addition work two addends are written at a given signal. From these seven, eight or ten addends are formed according to the instructions. Each new addend is formed by adding the two preceding ones. All the addends are then summed and when the total is correct the subtractor proceeds and subtracts a given subtrahend five times in succession. He in turn is followed by the multiplier and the latter by the divider according to their respective assignments. The team finishing first is declared the winner. The mechanics for handling the situation is described in the article referred to.

It is possible to get the entire class interested in the relays and in each of the four fundamental operations any one of which lends itself to the contest idea. In a very short time you will find groups of pupils proficient in one or more of the fundamentals. The entire class is divided into teams of four with a captain for each team. Every member of the class can do one of the operations well and is therefore on a team. The class competition becomes so keen that a representative team is soon chosen together with an alternate team to represent the class. These are often replaced through failure to win from a challenging team of the class. A round robin is arranged for the classes in the building. The most successful of these teams represents the building. The next step is the inter-annex contest and this leads to an all school team ready to meet all comers from other schools in the city. The final contest is an intercity contest but plans for this are yet in the making.

The relay always serves as an excellent drill in the class room and for outside assignment as a very simple change in the end number of the first addend makes all of the examples different yet very easy for quick verification on the part of the teacher. The subtrahend may be chosen so as to have a definite relation to the addends. Likewise the divisor may be chosen so as to have some definite relation to the multipliers. It has been found to be better to use nine addends or twelve addends after the ten addend work has been developed. If we designate the two addends by x and y, we have the following new addends and their subtotals. A number of ingenious combinations may be made from these algebraic relations so as to get an answer quickly from the original addends. Note the formation of coefficients.

	Addends	Subtotals
(1)	x	x
(2)	y	$x + y$
(3)	$x + y$	$2x + 2y$
(4)	$x + 2y$	$3x + 4y$
(5)	$2x + 3y$	$5x + 7y$
(6)	$3x + 5y$	$8x + 12y$

(7)	$5x + 8y$	$13x + 20y$
(8)	$8x + 13y$	$21x + 33y$
(9)	$13x + 21y$	$34x + 54y$
(10)	$21x + 34y$	$55x + 88y$
(11)	$34x + 55y$	$89x + 143y$
(12)	$55x + 89y$	$144x + 232y$
(13)	$89x + 144y$	$234x + 376y$
(14)	$144x + 233y$	$378x + 699y$

In the following examples for relay work twelve addends are required and the subtrahend is used five times in succession.

(1)
73659
85673
30483032 sum
sub. 5196783
4499117 rem.
mul. 9, 7, 6
1700666226 prod.
div. 189
8998234 quot.

(2)
98967
83958
33729504 sum
sub. 5431896
6570024 rem.
mul. 9, 8, 7
3311292096 prod
div. 756
4380016 quot.

(3)
87657
98976
35585040 sum
sub. 5638972
7390180 rem.
mul. 9, 8, 6
3192557760 prod.
div. 864
3695090 quot.

(4)
89678
56783
26087288 sum
sub. 4658937
2792603 rem.
mul. 9, 8, 7
1407471912 prod.
div. 504
2792603 quot.

In these contests it will be found advantageous to use squared paper or square ruled blackboards.

Another highly interesting series of contests is known as the Scissors or Criss-cross. This may be applied to the fundamental operations involving integers, fractions or decimals in the same manner as the relay and can be used for horizontal as well as vertical addition and subtraction.

The Scissors Applied to Addition.

A team of two pupils is designated "A" and "B." A and B begin work simultaneously at a given signal. They receive their respective assignments on library cards and transcribe these to the blackboard and then "blaze away." A is assigned addends (1) and (2) whereas B is assigned addends (10) and (11). They add in the relay fashion forming addends by adding each addend to the preceding one until eight addends have been written. The eight addends are summed. A and B interchange places, check their partner's work and then write their sum under that of the partner. They proceed in the new place making new addends until six have been written, then these are summed. A and B again interchange places, check and add their own last total to the partner's total, observing that the grand totals agree. The two pair of initial addends are composed of different numbers as shown below:

A begins here				B begins here
(1 or x)	396	these two addends are assigned	693	(10 or w)
(2 " y)	487	" " " " "	784	(11 or z)
(3)	883		1477	(12)
(4)	1370		2261	(13)
(5)	2253		3738	(14)
(6)	3623		5999	(15)
(7)	5876		9737	(16)
(8)	9499		15736	(17)
B continues here				A continues here
(9)	24387	A & B interchange places, check	40425	(18)
(18)	40425	partner's work, transfer sums	24387	(9)
(19)	64812		64812	(24)
(20)	105237		89199	(25)
(21)	179049		154011	(26)
(22)	275286		243210	(27)
(23)	680196	A & B interchange places, check	616044	(28)
A resumes here				B resumes here
(28)	616044	partner's work, transfer sums	680196	(23)
(30)	1296240	and add	1296240	(29)

The answer always ends in a cipher and is 20 times the sum of (9) and (18). An increase of 1 in either (1) or (10) increases the answer by 420. An increase of 1 in either (2) or (11) increases the answer by 660. The final sum equals 420 $(x+w)+660\ (y+z)$. Therefore if x is increased by the same amount which is subtracted from w the answer remains unchanged. This is likewise true for the combination $y+z$. It is this observation that enables us to set any number of addend pairs which will produce a given result, obviating copying of work by pupils.

If the addends are chosen so that $(x+w)=1000$ and also $(y+z)=1000$ the answer reduces to 1080000 which may be used as a key by means of which a number of examples may be set without effort as shown below:

$$\begin{array}{llll} x\ 306 & 694\ w & x+w=1000 & \\ y\ 487 & 513\ z & y+z=1000 & \text{sum}=1080000 \end{array}$$

If the addends are chosen so as to form pairs of complementary numbers which are multiples of 100 another set of examples may be formed as follows:

$$\begin{array}{ll} x\quad 396 & 693\ w \\ x+p\ 404 & 707\ w+q \\ & \text{sum}=1080(x+w)+660(p+q)=1190640 \end{array}$$

If $(x+w)=1000$ then this reduces to $1080000+660\ (p+q)$ as shown below:

$$\begin{array}{lll} x\quad 396 & 604\ w & x+w=1000 \\ x+p\ 404 & 796\ w+q & p+q\ =8+192=200 \\ & \multicolumn{2}{l}{\text{sum}=1080000+660(200)=1212000} \end{array}$$

The following examples are appended for reference:
Examples whose sum = 1080000:

396	604	396	604	397	693	395	605
487	513,	488	512,	489	511,	480	520,

Examples whose sum = 1080660:

396	604	396	604	397	693	395	605
487	514,	488	513,	489	512,	480	521,

Examples whose sum = 1298400:

397	694	396	695	398	693	397	694
488	785,	487	786,	489	784,	487	786,

Examples whose sum = 1299060:

415	676	415	676	417	674	416	675
489	785	488	786,	488	786,	487	787,

Examples whose sum = 3459060:

1415	1676	1415	1676	1417	1674	1416	1675
1489	1785,	1488	1786,	1488	1786,	1487	1787,

Examples whose sum = 345906:

141.5	167.6	141.5	167.6	141.7	167.4
148.9	178.5.	148.8	178.6,	148.8	178.6,

It may be possible to prevail upon the editors of the Mathematics Teacher to give space for the publication of the activities of an intercity mathematics club. This space could be devoted to the activities of mathematics clubs, notices of contests and results. Under the auspices of the mathematics teacher and the local school organizations an intercity contest could be conducted by telephone or by having teams visit other city teams. The pleasure attending such a visit would prove a desirable incentive for a very active participation in the tryout contests. Arrangements could be made so that the expense of such a trip would not exceed the carfare for travel. The members of visiting teams would be distributed and entertained by the teachers of that school which acts as host.

"In their games children are actors, architects, and poets, and sometimes musical composers as well."

High School of Commerce,
New York City.